

Superluminal neutrinos in Hořava-Lifshitz gravity

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We present a possible explanation of the OPERA results on the superluminal muon neutrinos, by examining the fermion propagation in the earth's gravitational field, considering the gravitational sector to be of Hořava-Lifshitz type. In particular, investigating the Dirac equation in the spherical solutions of the theory, we find that the neutrinos feel an effective metric with respect to which they propagate superluminally. This is not the case in larger distances, where standard cosmological geometry is restored.

PACS numbers: 04.60.Bc, 04.50.Kd, 13.15.+g

I. INTRODUCTION

Almost ten days ago the OPERA collaboration announced the astonished result that muon-neutrinos created in CERN CNGS beam in Geneva were detected in Sasso Laboratory in central Italy, faster than the time light would need to cover the same distance in vacuum [1]. In particular, the ν_μ velocity v was reported to be

$$(v - c)/c = (2.48 \pm 0.28(\text{stat}) \pm 0.30(\text{sys})) \times 10^{-5}, \quad (1)$$

a result that is in agreement with earlier 1σ MINOS announcements [2]. Although the possibility of, unknown up to now, systematic errors is still a reasonable explanation, the above measurement attracted the interest of theorists, who try to explain it following many different paths: in [3–5] proposing simple models of Lorentz violation by hand, in [6] imposing a mass-dependent Lorentz violation, in [7] using models of energy-dependent velocities, in [8] with a Fermi-point splitting, in [9] imposing Lorentz violations through light sterile neutrinos, in [10–12] assuming that neutrinos can propagate in extra dimensions, in [13] using Finsler spacetimes, in [14] using Gödel-like rotating universe, in [15] using tachyonic mixed neutrinos (although in [16] it was shown that superluminality cannot be explained using tachyonic or Coleman-Glashow models), in [17] using a domain wall, in [18] using a neutrino-scalar coupling, in [19] using the quantum Hamilton-Jacobi equation, and in [20] through a truer measurement of Einstein's limiting speed. Additionally, in [21] and [22] a Lifshitz-type, Lorentz-violated fermion model was assumed, while in [23] and [24] the authors proposed the interesting idea of a local effective neutrino supelumination, without Lorentz violation, due to a coupling with a new spin-2 field or a scalar respectively. Finally, we have to mention that there are works claiming that the superluminal interpretation is not correct: in [25, 26] it was argued that the superluminal neutrinos would decay through a number of channels, while in [27] the author put into question the convention for synchronization of clocks in non-inertial frames.

In the present work we propose an explanation of the OPERA results investigating the neutrino propagation in the earth's gravitational field, in the context of Hořava-Lifshitz gravity. In particular, we argue that the neutrinos are superluminal with respect to the effective background metric, which is not the case in larger distances where standard cosmological geometry is restored.

II. SPHERICAL SOLUTIONS IN HOŘAVA-LIFSHITZ GRAVITY

Let us briefly review the spherical solutions of simple Hořava-Lifshitz gravity. The dynamical variables are the lapse and shift functions, N and N_i respectively, and the spatial metric g_{ij} (roman letters indicate spatial indices). In terms of these fields the full metric is written as $ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$, and the (anisotropic) scaling transformation of the coordinates reads: $t \rightarrow l^3 t$ and $x^i \rightarrow l x^i$. As it is known, the action of the theory can be decomposed as [28, 29]

$$S = \int dt d^3x \sqrt{g} N (\mathcal{L}_0 + \mathcal{L}_1) \quad (2)$$

$$\mathcal{L}_0 = \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda R - 3\Lambda^2)}{8(3\lambda - 1)} \quad (3)$$

$$\mathcal{L}_1 = \frac{\kappa^2}{2w^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu}{2w^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R_k^l + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} - \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{32(3\lambda - 1)} R^2, \quad (4)$$

where $K_{ij} = (g_{ij} - \nabla_i N_j - \nabla_j N_i)/2N$ is the extrinsic curvature and $C^{ij} = \epsilon^{ijk} \nabla_k (R_i^j - R \delta_i^j/4)/\sqrt{g}$ the Cotton tensor, and the covariant derivatives are defined with respect to the spatial metric g_{ij} . ϵ^{ijk} is the totally antisymmetric unit tensor, κ , w , μ and Λ are constants (we have already performed the usual analytic continuation of the parameters μ and w and thus Λ is positive), and λ is the dimensionless constant that determines the flow between the IR and UV. We mention that in writing the above action splitting, and with the particular coefficients, we have imposed the detailed balance condition [28], which allows for a quantum inheritance principle

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[29], since the $(D + 1)$ -dimensional theory acquires the renormalization properties of the D -dimensional one. Finally, it is straightforward to see that for the light speed, the gravitational Newton's constant¹ and the effective cosmological constant we obtain:

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda}{3\lambda - 1}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda_{eff} = \frac{3\kappa^2 \mu^2 \Lambda^2}{16(3\lambda - 1)}. \quad (5)$$

As one observes, the light speed flows too, however one can still set it to 1, and consider photons to propagate with this speed always, which will be the reference speed in Hořava-Lifshitz gravity.

Under different assumptions there are many spherical solutions in the gravitational scenario at hand [31–34], which extract the extra terms comparing to General Relativity. For the purpose of this work we desire to remain in a general but still simple level. Thus, we should go beyond the detailed balance condition, which proves to lead to theoretical and observational problems [35, 36], but still keeping the structure of the theory simple. Therefore, it is adequate to deform action (2) as [31, 32]

$$S = \int dt d^3x \sqrt{g} N \{ \mathcal{L}_0 + (1 - \epsilon^2) \mathcal{L}_1 \}, \quad (6)$$

with ϵ a parameter.

Seeking for static, spherically symmetric solutions with the metric ansatz

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (7)$$

and setting $\lambda = 1$, as expected for earth scales, one obtains:

$$\begin{aligned} N(r)^2 &= f(r) \\ &= 1 + \frac{\Lambda r^2}{1 - \epsilon^2} - \frac{\sqrt{\alpha^2(1 - \epsilon^2)\sqrt{\Lambda}r + \epsilon^2\Lambda^2 r^4}}{1 - \epsilon^2}. \end{aligned} \quad (8)$$

In this expression the integration constant α can be expressed in terms of the total mass of the spherical object and the gravitational Newton's constant [31, 32].

We mention here that the peculiar second term in (8), which will play the central role in the following discussion, arises in the majority of the corresponding solutions [31–34]. For example, if we take the Kehagias-Sfetsos (KS) model [34] and extend it to the minimal beyond-detailed-balance case (that is taking a general coefficient of the Ricci scalar term in the action (2)) we can obtain the KS spherical solution

$$N_{KS}(r)^2 = f_{KS}(r) = 1 + qr^2 - \sqrt{r(q^2 r^3 + 4qMG)}, \quad (9)$$

where q is now a free parameter, positive due to the analytic continuation, and M is the total mass (note that one could alternatively obtain the above solution keeping the detailed-balance version of KS model, but move slightly away from $\lambda = 1$).

III. NEUTRINOS MOTION IN EARTH'S GRAVITATIONAL FIELD

Let us now investigate the propagation of fermions, and in particular of neutrinos, in earth's gravitational field. Considering a massive Dirac field in a curved background $g_{\mu\nu}$, the equation of motion reads [37]:

$$\left[\gamma^a e_a^\mu (\partial_\mu + \Gamma_\mu) + \frac{m}{\hbar} \right] \Psi = 0, \quad (10)$$

where m is the fermion mass and \hbar the Planck's constant. In this relation e_a^μ is the inverse of the vierbein tetrad field e_μ^a , defined as $g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b$ with $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$, γ^a are the Dirac matrices (taken in the standard representation [37]) and Γ_μ is the spin connection given by

$$\Gamma_\mu = \frac{1}{8} [\gamma^a, \gamma^b] e_a^\nu e_{b\nu;\mu}, \quad (11)$$

where the covariant derivative of $e_{b\nu}$ is as usual $e_{b\nu;\mu} = \partial_\mu e_{b\nu} - \Gamma_{\mu\nu}^a e_{ba}$.

Let us investigate the Dirac equation in the earth's background, considering that its gravitational field is given by (7), that is in a vierbein reading as

$$e_a^\mu = \text{diag}\left(\frac{1}{\sqrt{f}}, \sqrt{f}, \frac{1}{r}, \frac{1}{r \sin \theta}\right), \quad (12)$$

with $f(r)$ given by (8) or (9). Neglecting for simplicity the spin connection Γ_μ , which proves to be very small, the Dirac equation (10) under the geometry (12) reads:

$$\left(\frac{\gamma^0}{\sqrt{f(r)}} \partial_t + \sqrt{f(r)} \gamma^1 \partial_r + \frac{\gamma^2}{r} \partial_\theta + \frac{\gamma^3}{r \sin \theta} \partial_\phi + \frac{m}{\hbar} \right) \Psi = 0. \quad (13)$$

From this relation one can immediately see that the neutrinos feel an effective metric, and that their velocity is simply

$$v(r) = f(r), \quad (14)$$

and in particular if they propagate in an approximately constant r , equal for instance with the earth's radius $r = R_\oplus$, their speed will be $v = f(R_\oplus) = \text{const.}$

Additionally, we can verify this result by approximately solving the Dirac equation (13) under certain assumptions. In particular, in the standard Dirac matrices representation the fermion wave function is written as

$$\Psi(t, r, \theta, \phi) = \begin{pmatrix} A(t, r, \theta, \phi) \\ 0 \\ B(t, r, \theta, \phi) \\ 0 \end{pmatrix} \exp \left[\frac{i}{\hbar} I(t, r, \theta, \phi) \right]. \quad (15)$$

¹ Note that in theories with Lorentz invariance breaking the “gravitational” Newton's constant, that is the one that is read from the action, does not coincide with the “cosmological” Newton's constant, that is the one that is read from the Friedmann equations [30], but this is irrelevant for the purposes of this work where we focus on non-cosmological scales.

Without loss of generality, and in order to avoid difficulties of solving Dirac equation in spherical coordinates, and using the spherical symmetry, we can assume that A and B are constants, while $I(t, r, \theta, \phi) = -\omega t + p(r)r + \Theta(\theta, \phi)$ [38], with $p(r)$ the neutrino momentum. In such a case the two relevant equations read

$$\begin{aligned} -\frac{A}{\sqrt{f(r)}}\omega + B\sqrt{f(r)}p(r) + mA &= 0 \\ \frac{B}{\sqrt{f(r)}}\omega - A\sqrt{f(r)}p(r) + mB &= 0, \end{aligned} \quad (16)$$

and thus the solution condition (the determinant of A, B coefficients to be zero) leads to the dispersion relation

$$\omega^2 = f(r)^2 p(r)^2 + m^2 f(r). \quad (17)$$

In the massless case we can see that both the group velocity $\partial\omega/\partial p$ and the phase velocity ω/p are equal to f , that is $v(r) = f(r)$.

In summary, we showed that the neutrino's velocity in the earth's gravitational field in Hořava-Lifshitz gravity is equal to $v = f(r)$, with $f(r)$ given by (8) or (9) according to the specific solution subclass one uses. Observing the form of $f(r)$ we can clearly see that $v(r)$ becomes superluminal, that is $v(r) > 1$, for distances larger than a specific value. Clearly this is not the case in General Relativity spherical solutions, where the examination of the Dirac equation, similarly to the above procedure, leads always to $v < 1$ (the second term in the metrics (8), (9) is absent).

Let us now come to the OPERA experiment. Since the neutrino motion takes place approximately on earth's surface, we deduce that the neutrinos have a constant velocity $v = f(R_\oplus)$. Thus, if we want this to be superluminal by an amount of 10^{-5} , we deduce that the parameter ϵ in solution subclass (8) must be of the order of $1 - \epsilon^2 \approx 10^{-15}$ (we use relations (5) in order to set c and G to 1 and then we use the values of Λ_{eff} , R_\oplus and M_\oplus in these units). Similarly, for the Kehagias-Sfetsos solution subclass we can see that the observed superluminality is obtained for $q \approx 10^{-18}$.

IV. DISCUSSION

In the present work we tried to explain the results of the OPERA collaboration on the superluminal muon neutrinos, by examining the fermion propagation in the earth's gravitational field, considering the gravitational sector to be of Hořava-Lifshitz type. In particular, we used the spherical solutions of the theory going beyond the detailed-balance condition, and in such a background we investigated the Dirac equation. We found that the neutrinos feel an effective metric with respect to which they propagate superluminally, and indeed if the detailed-balance condition is broken, the OPERA results can be explained. The reason for such a behavior is that

in spherical Hořava-Lifshitz solutions one obtains an extra positive term in the effective metric, and subsequently in the fermion velocity. In general, such a result is expected for Lifshitz-type theories and it plays the role of the “anti-gravity” source that is needed for superluminality, and indeed our own result in the specific case of Hořava-Lifshitz gravity is in agreement with the general qualitative result of [21, 22].

However, there are still some points that need to be clarified if we desire such a scenario to be the explanation of OPERA experiments. The first and obvious question is what can be said about neutrinos coming from galactical distances. In particular, anti-neutrino observations from the SN1987A supernova impose the stringent constraint $|(v - c)/c| < 2 \times 10^{-9}$ [39–41]². This result can be qualitatively explained in the present model, since away from the earth's surface the background metric is not spherical and it is not determined by the earth anymore, but from the sun, the other planets, the other stars etc, resulting to the Friedmann-Robertson-Walker metric, where the above procedure results to luminal speed for massless neutrinos. Definitely, a more precise and quantitative description is needed. However, note that there is an alternative explanation, namely that photons feel the effective metric too, and thus their own speed is also increased with respect to the laboratory-based, vacuum measurement. In this case the present model would explain simultaneously OPERA and SN1987A results.

The second point is what version of Hořava-Lifshitz gravity must be used, and which solution subclass. For example one can see that detailed-balance versions lead always to subluminal velocities. In the present work we desired to provide two examples where superluminality is possible in Hořava-Lifshitz context. Thus, we chose a simple version of Hořava-Lifshitz gravity, allowing also from a departure from the detailed-balance condition, as a representative example, despite the fact that more complicated extensions seem to be theoretically more robust [43]. Clearly, one should repeat the above procedure for such modified theories, however the complication of the scenario does not allow even for an acceptable examination of general spherical solutions. Finally, even for a given extended version of the theory, one may have many spherical solution subclasses, and the examination of all of them, or the question of which subclass should be expected for earth's field, is an important point which lies beyond the scope of the present work.

The third point is related to the Solar-system tests, and specifically to the Parametrized-Post-Newtonian (PPN) parameters of a given gravitational theory [44]. In particular, in the present model we demand the earth's gravita-

² However, note the interesting argument that probably the neutrino superluminality may indeed be the case in galactical and cosmological distances, serving as the explanation for the unsuccessful observation of neutrinos emitted in coincidence with the Gamma-Ray Burst [42].

tional field to deviate from the General Relativity prediction by approximately one part out of 10^{15} , in order for the OPERA-observed superluminality to be explained. However, such a small deviation from General Relativity lies beyond the limits of the measured values (the relevant PPN parameters are $\beta - 1 = 2.3 \times 10^{-5}$ and $\gamma - 1 = 4 \times 10^{-4}$ [44]), and thus a Hořava-Lifshitz-type gravitational theory is still allowed.

There are two more points related to the particle physics point of view. The first arises from the fact that strictly speaking the neutrinos arise in doublets with their accompanied leptons, and thus the above procedure would lead to superluminal muons or electrons too. However, at present there is no experiment which allows for these leptons to travel to such large distances as neutrinos did, and thus such an interesting possibility remains open. The second point is related to neutrino oscillations. Indeed one should try to incorporate this behavior in a quantitative way in the present scenario, and furthermore to focus on the possibility of one or more flavors, specifically ν_μ , to be massive. Such studies could be crucial for verifying or ruling out the present scenario.

Finally, we mention here that the model at hand has two predictions that are distinguishable from other works

that try to explain the OPERA experiment. The first is that the neutrino superluminality depends only on r , namely the distance from earth's center, and not on the neutrinos' traveled distance (as long as it remains approximately on the surface). Therefore, detection in double or triple distances would lead to the same velocity result. The second prediction of the present work is that the neutrino's velocity does not depend on the energy, which indeed seems to be the case in OPERA ($\delta t = (53.1 + 18.8_{\text{stat}})\text{ns}$ for $\langle E_\nu \rangle = 13.9\text{GeV}$ and $\delta t = (67.1 + 18.2_{\text{stat}})\text{ns}$ for $\langle E_\nu \rangle = 42.9\text{GeV}$). However, a larger energy-interval should be scanned before one can make a safe statement.

In summary, superluminal fermion propagation, if verified, opens the way to many novel scenarios, and one of them proposed here is a Hořava-Lifshitz gravitational theory.

Acknowledgments

The author wishes to thank Alex Kehagias for useful discussions.

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